Casimir effect for thin films from imperfect materials.

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We propose an approach for investigation of interaction of thin material films with quantum electrodynamic fields. Using main principles of quantum electrodynamics (locality, gauge invariance, renormalizability) we construct a single model for Casimir-like phenomena arising near the film boundary on distances much larger then Compton wavelength of the electron where fluctuations of Dirac fields are not essential. In this model the thin film is presented by a singular background field concentrated on a 2-dimensional surface. All properties of the film material are described by one dimensionless parameter. For two parallel plane films we calculate the photon propagator and the Casimir force, which appears to be dependent on film material and can be both attractive and repulsive. We consider also an interaction of plane film with point charge and straight line current. Here, besides usual results of classical electrodynamics the model predicts appearance of anomalous electric and magnetic fields.

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It is well known that the state of quantum field theoretical system is strongly influenced by external background. On the other hand, the fluctuations of quantum fields make essential correction in a classical picture of interaction for material objects. In 1948, Casimir showed that there is a generated attraction between two parallel uncharged planes [1]. Theoretical prediction of this phenomena called the Casimir effect (CE) has been well confirmed with modern experimental dates [2, 3, 4]. Nowadays, the CE appears to be of practical importance because the quantum phenomena of such kind can be essential for micro-mechanics and nano-technology.

There are many theoretical results concerning the CE (see for example recent review [5]). However, many authors being interested in some particular aspects of the CE only, prepare calculations in the framework of simplified models. Usually, it is supposed that the specifics of quantum electrodynamics (QED) are not significant, and most essential features of the CE can be investigated in the framework of free quantum scalar field theory with fixed boundary conditions or δ-function potentials [6, 7]. By means of such methods one can obtain quantitative description of some of the CE characteristics, but there is no possibility of studying other phenomena generated by interaction of the QED fields with classical background within the same model.

In this paper we propose a single model for investigation of all peculiar properties of the CE for thin material films. Here the film is presented by a singular background (defect) concentrated on the 2-dimensional surface. Its interaction with a photon field appears to be completely defined by the geometry of the defect and restrictions following from the basic principles of QED (gauge invariance, locality, renormalizability). The locality of interaction means that the action functional of the defect is represented by an integral over defect surface of the Lagrangian density which is a polynomial function of space-time point in respect to fields and derivatives of ones. The coefficients of this polynomial are the parameters defining defect properties. The requirement of renormalizibility sets strong restrictions on form of defect action in quantum field theory (QFT). The first analysis of renormalisation problem in local QFT with defects has been made by Symanzik in [8]. Symanzik showed that in order to keep renormalizability of the bulk QFT, one needs to add a defect action to the usual bulk action of QFT model. The defect action must contain all possible terms with nonnegative dimensions of parameters and not include any parameters with negative dimensions. In case of QED the defect action must be also gauge invariant.

From these requirements it follows that for thin film (without charges and currents) which shape is defined by equation $\Phi(x) = 0$, $x = (x_0, x_1, x_2, x_3)$, the action describing its interaction with photon field $A_\mu(x)$ reads

$$S_\Phi(A) = \frac{a}{2} \int \varepsilon^{\lambda\mu\nu\rho} \partial_\lambda \Phi(x) A_\mu(x) F_{\nu\rho}(x) \delta(\Phi(x)) dx \quad (1)$$

where $F_{\nu\rho}(x) = \partial_\nu A_\rho - \partial_\rho A_\nu$, $\varepsilon^{\lambda\mu\nu\rho}$ denotes totally antisymmetric tensor ($\varepsilon^{0123} = 1$), a is a constant dimensionless parameter. Expression (1) is the most general form of gauge invariant action concentrated on the defect surface being invariant in respect to reparametrization of one and not having any parameters with negative dimensions.

The CE-like phenomena considered directly on the distances from the defect boundary much larger then Compton wavelength of the electron are not influenced essentially by the Dirac fields in QED because of exponential damping of fluctuations of those on much smaller distances ($\sim m_e^{-1} \approx 10^{-10} cm$ for electron, $\sim m_p^{-1} \approx 10^{-13} cm$ for proton) [9]. Thus, by theoretical studies of CE one can neglect the Dirac fields and use the model of quantum electromagnetic field (photodynamic) with ad-

ditional defect action (1). Such models are considered in this paper.

In order to expose some features of ones, we calculate the Casimir force (CF) in the model for two parallel infinite plane films. We consider also an interaction of the plane film with a parallel to it straight line current and an interaction of film with a point charge. For these systems we calculate electric and magnetic fields.

The quantitative description of all physical phenomena caused by interaction of the film with the photon field and classical charges can be obtained if the generating functional of Greens functions is known. For gauge condition $\phi(A)=0$ it is of the form

$$G(J) = C\int e^{iS(A,\Phi)+iJA}\delta(\phi(A))DA \tag{2}$$

where

$$S(A,\Phi) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + S_\Phi(A), \tag{3}$$

and the constant C is defined by normalization condition $G(0)|_{a=0} = 1$, i.e. in pure photodynamic without defect $\ln G(0)$ vanishes. The first term on the right hand side of (3) is the usual action of photon field. Along with defect action it forms a quadratic in photon field full action of the system which can be written as $S(A,\Phi) = 1/2\ A_\mu K_\Phi^{\mu\nu} A_\nu$. The integral (2) is gaussian and is calculated exactly:

$$G(J) = \exp\left\{\frac{1}{2}Tr\ln(D_\Phi D^{-1}) - \frac{1}{2}JD_\phi J\right\}$$

where D_Φ is the propagator $D_\Phi = iK_\Phi^{-1}$ of photodynamic with defect in gauge $\phi(A)=0$, and D is the propagator of photon field without defect in the same gauge. For the static defect, function $\Phi(x)$ is time independent, and $\ln G(0)$ defines the Casimir energy.

To take into account effects of finite width one should add to (3) bulk defect action with a set of independent material parameters. As we show below bulk contribution to Casimir energy can be neglected for thin films.

We restrict ourselves with consideration of the simplest case of plane infinite films. We begin with defect concentrated on two parallel planes $x_3=0$ and $x_3=r$. For this model, it is convenient to use a notation like $x=(x_0,x_1,x_2,x_3)=(\vec{x},x_3)$. Defect action (1) has the form:

$$S_{2P} = \frac{1}{2}\int (a_1\delta(x_3) + a_2\delta(x_3-\mathrm{r}))\varepsilon^{3\mu\nu\rho}A_\mu(x)F_{\nu\rho}(x)dx.$$

The defect action S_{2P} was discussed in (10) in substantiation of Chern-Simon type boundary conditions chosen for studies of the Casimir effect in photodynamics [10]. This approach based on boundary conditions is not related directly to the one we present. The defect action (1) is the main point in our model formulation, and no any boundary conditions are used. The action S_{2P} is translationaly invariant with respect to coordinates x_i, $i=0,1,2$. The propagator $D_\Phi(x,y)$ is written as:

$$D_{2P}(x,y) = \frac{1}{(2\pi)^3}\int D_{2P}(\vec{k},x_3,y_3)e^{i\vec{k}(\vec{x}-\vec{y})}d\vec{k},$$

and $D_{2P}(\vec{k},x_3,y_3)$ can be calculated exactly. Using latin indexes for the components of 4-tensors with numbers $0,1,2$ and notations

$$P^{lm}(\vec{k}) = g^{lm} - k^l k^m/\vec{k}^2, \quad L^{lm}(\vec{k}) = \epsilon^{lmn3}k_n$$

(g is metrics tensor) one can present the results for the Coulomb-like gauge $\partial_0 A^0 + \partial_1 A^1 + \partial_2 A^2 = 0$ as follows

$$\begin{aligned}
D^{lm}_{2P}(\vec{k},x_3,y_3) = \frac{P^{lm}(\vec{k})}{2|\vec{k}|}\Bigg\{ &\frac{(a_1a_2 - a_1^2a_2^2(1-e^{2i|\vec{k}|\mathrm{r}}))(e^{i|\vec{k}|(|x_3|+|y_3-\mathrm{r}|)} + e^{i|\vec{k}|(|x_3-\mathrm{r}|+|y_3|)})e^{i|\vec{k}|\mathrm{r}}}{[(1+a_1a_2(e^{2i|\vec{k}|\mathrm{r}}-1))^2 + (a_1+a_2)^2]} + \\
&+\frac{(a_1^2 + a_1^2a_2^2(1-e^{2i|\vec{k}|\mathrm{r}}))e^{i|\vec{k}|(|x_3|+|y_3|)} + (a_2^2 + a_1^2a_2^2(1-e^{2i|\vec{k}|\mathrm{r}}))e^{i|\vec{k}|(|x_3-\mathrm{r}|+|y_3-\mathrm{r}|)}}{[(1+a_1a_2(e^{2i|\vec{k}|\mathrm{r}}-1))^2 + (a_1+a_2)^2]} - e^{i|\vec{k}||x_3-y_3|}\Bigg\} - \\
-\frac{L^{lm}(\vec{k})}{2|\vec{k}|^2[(1+a_1a_2(e^{2i|\vec{k}|\mathrm{r}}-1))^2 + (a_1+a_2)^2]} &\Big\{a_1a_2(a_1+a_2)\left(e^{i|\vec{k}|(|x_3|+|y_3-\mathrm{r}|)} + e^{i|\vec{k}|(|x_3-\mathrm{r}|+|y_3|)}\right)e^{i|\vec{k}|\mathrm{r}} - \\
&-a_1(1+a_2(a_2+a_1e^{2i|\vec{k}|\mathrm{r}}))\ e^{i|\vec{k}|(|x_3|+|y_3|)} - a_2(1+a_1(a_1+a_2e^{2i|\vec{k}|\mathrm{r}})e^{i|\vec{k}|(|x_3-\mathrm{r}|+|y_3-\mathrm{r}|)}\Big\},
\end{aligned}$$

$$D^{l3}_{2P}(\vec{k},x_3,y_3) = D^{3m}_{2P}(\vec{k},x_3,y_3) = 0, \quad D^{33}_{2P}(\vec{k},x_3,y_3) = \frac{-i\delta(x_3-y_3)}{|\vec{k}|^2}, \quad |\vec{k}| \equiv \sqrt{k_0^2 - k_1^2 - k_2^2}.$$

The energy density E_{2P} of defect is defined as

$$\ln G(0) = \frac{1}{2}\ \mathrm{Tr}\ln(D_{2P}D^{-1}) = -iTSE_{2P}$$

where $T = \int dx_0$ is time of existence of defect, and $S =$

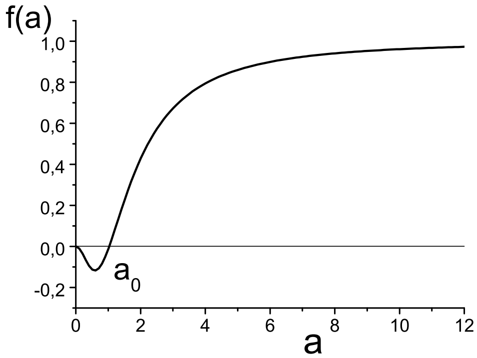

FIG. 1: Function $f(a)$ determining Casimir force between parallel planes

$\int dx_1 dx_2$, is area of one. It is expressed in an explicit form in terms of polylogarithm function $\mathrm{Li}_4(x)$ in the following way:

$$E_{2P} = \sum_{j=1}^{2} E_j + E_{Cas}, E_j = \frac{1}{2}\int \ln(1+a_j^2)\frac{d\vec{k}}{(2\pi)^3}, j=1,2,$$

$$E_{Cas} = -\frac{1}{16\pi^2 \mathrm{r}^3}\sum_{k=1}^{2} \mathrm{Li}_4\left(\frac{a_1 a_2}{a_1 a_2 + i(-1)^k(a_1+a_2)-1}\right)$$

Here E_j is an infinite constant, which can be interpreted as self-energy density on the j-th planes, and E_{Cas} is an energy density of their interaction. Function $\mathrm{Li}_4(x)$ is defined as

$$\mathrm{Li}_4(x) = \sum_{k=1}^{\infty}\frac{x^k}{k^4} = -\frac{1}{2}\int_0^{\infty} k^2 \ln(1-xe^{-k})dk.$$

For identical defect planes ($a_1 = a_2 = a$) the force $F_{2P}(\mathrm{r}, a)$ between them is given by

$$F_{2P}(\mathrm{r}, a) = -\frac{\partial E_{Cas}(\mathrm{r}, a)}{\partial \mathrm{r}} = -\frac{\pi^2}{240\mathrm{r}^4} f(a).$$

Function $f(a)$ is plotted on the Fig. 1. It is even ($f(a) = f(-a)$) and has a minimum at $|a| = a_m \approx 0.5892$ ($f(a_m) \approx 0,11723$), $f(0) = f(a_0) = 0$ by $a_0 \approx 1,03246$, and $\lim_{a\to\infty} f(a) = 1$. For $0 < a < a_0$ ($a > a_0$), function $f(a)$ is negative (positive). Therefore the force F_{2P} is repulsive for $|a| < a_0$ and attractive for $|a| > a_0$. For large $|a|$ it is the same as the usual CF between perfectly conducting planes. This model predicts that the maximal magnitude of the *repulsive* F_{2P} (about $0,1$ of the CF's magnitude for perfectly conducting planes) is expected for $|a| \approx 0.6$. For two infinitely thick parallel slabs the repulsive CF was predicted also in [11].

In real physical situations the film has a finite width, and the bulk contributions to the CF for nonperfectly conducting slabs with widths h_1, h_2 are proportional to $h_1 h_2$. Therefore it follows directly from the dimensional analysis that the bulk correction F_{bulk} to the CF is of the form $F_{bulk} \approx cF_{Cas}h_1 h_2/r^2$ where F_{Cas} is the CF for perfectly conducting planes and c is a dimensionless constant. This estimation can be relevant for modern experiments on the CE. For instance, in [4] there were results obtained for parallel metallic surfaces where width of layer was about $h \approx 50$ nm and typical distance r between surfaces was $0.5\mu m \leq r \leq 3\mu m$. In that case $3\times 10^{-4} \leq (h/r)^2 \leq 10^{-2}$. In [4] authors have fitted the CF between chromium films with function C_{Cas}/r^4. They claim that the value of C_{Cas} coincides with known Casimir result within a 15% accuracy. It means that bulk force can be neglected, and only surface effects are essential. In our model the values $a > 4.8$ of defect coupling parameter a are in good agreement with results of [4].

To reveal some specific effects generated by interaction of the film with the photon field within the proposed model, one can study the scattering of electromagnetic waves on the plane and coupling of plane with a given classical 4-current.

For the first problem one can use a homogeneous classical equation $K_{2P}^{\mu\nu}A_\nu = 0$ of simplified model with $a_1 = a$, $a_2 = 0$. It has a solution in the form of a plane wave. If one defines transmission (reflection) coefficient as a ratio of transmitted U_t (reflected U_r) energy to incident U_{in} one: $K_t = U_t/U_{in}$, ($K_r = U_r/U_{in}$), then direct calculations give the following result: $K_t = (1+a^2)^{-1}, K_r = a^2(1+a^2)^{-1}$. We note two features of reflection and transmission coefficients. The first one is that in the limit of infinitely large defect coupling these coefficients coincide with coefficients for a perfectly conducting plane. The second one is that they do not depend on the incidence angle, and that can be attributed to $\epsilon\mu = 1$ condition discussed below.

The classical charge and the wire with current near defect plane are modeled by appropriately chosen 4-current J in (2). The mean vector potential $\mathcal{A}_\mu$ generated by J and the plane $x_3 = 0$, with $a_1 = a$ can be calculated as

$$\mathcal{A}^\mu = -i\frac{\delta G(J)}{\delta J_\mu}\bigg|_{a_1=a, a_2=0} = iD_{2P}^{\mu\nu}J_\nu|_{a_1=a, a_2=0}. \qquad (4)$$

Using notations $\mathcal{F}_{ik} = \partial_i\mathcal{A}_k - \partial_k\mathcal{A}_i$, one can present electric and magnetic fields as $\vec{E} = (\mathcal{F}_{01}, \mathcal{F}_{02}, \mathcal{F}_{03})$, $\vec{H} = (\mathcal{F}_{23}, \mathcal{F}_{31}, \mathcal{F}_{12})$. For charge e at the point $(x_1, x_2, x_3) = (0, 0, l)$, $l > 0$ the corresponding classical 4-current is

$$J_\mu(x) = 4\pi e\delta(x_1)\delta(x_2)\delta(x_3 - l)\delta_{0\mu}$$

In virtue of (4), the electric field in considered system is the same as one generated in usual classical electrostatic by charge e placed on distance l from infinitely thick slab with dielectric constant $\epsilon = 2a^2 + 1$. The defect plane

induces also a magnetic field $\vec{H} = (H_1, H_2, H_3)$:

$$H_1 = \frac{eax_1}{(a^2+1)\rho^3},\ H_2 = \frac{eax_2}{(a^2+1)\rho^3},\ H_3 = \frac{ea(|x_3|+l)}{(a^2+1)\rho^3}.$$

where $\rho = (x_1^2 + x_2^2 + (|x_3|+l)^2)^{\frac{1}{2}}$. It is an anomalous field which doesn't arise in classical electrostatics. Its direction depends on sign of a. A current with density j flowing in the wire along the x_1-axis is modeled by

$$J_\mu(x) = 4\pi j\delta(x_3 - l)\delta(x_2)\delta_{\mu 1}$$

For magnetic field from (4) we obtain usual results of classical electrodynamics for the current parallel to infinitely thick slab with permeability $\mu = (2a^2+1)^{-1}$. There is also an anomalous electric field $\vec{E} = (0, E_2, E_3)$:

$$E_2 = \frac{2ja}{a^2+1}\frac{x_2}{\tau^2},\ E_3 = \frac{2ja}{a^2+1}\frac{|x_3|+l}{\tau^2}$$

where $\tau = (x_2^2 + (|x_3|+l)^2)^{\frac{1}{2}}$. Comparing both formulae for parameter a we obtain the relation $\epsilon\,\mu = 1$. It holds for material of thick slab interaction of which with point charge and current in classical electrodynamics was compared with results for thin film of our model. The speed of light in this hypothetical material is equal to one in the vacuum. From the physical point of view, it could be expected, because interaction of film with photon field is a surface effect which can not generate the bulk phenomena like decreasing the speed of light in the considered slab.

The relation $\epsilon\,\mu = 1$ is not new in the context of the Casimir theory. It was first introduced by Brevik and Kolbenstvedt [12] who calculated the Casimir surface force density on the sphere. Only on this condition a contact term turn out to be zero [12]. It has been investigated in a number of subsequent papers. In our approach this condition arises naturally because we have only one parameter a that must describe both magnetostatic and electrostatic properties of the film. It is quite possible that macroscopic properties of surface are different as bulk ones due to the loss of periodicity of lattice potential in one dimension. This happens because the presence of surface leads to a change of spectrum of electron states on and near the surface [13]. The essential property of interaction of films with classical charge and current is the appearance of anomalous fields. This fields are suppressed in respect of usual ones by factor a^{-1} and they vanish in case of perfectly conducting plane. Magnetoelectric (ME) two-dimensional materials [14] are good candidates to detect anomalous fields and non ideal CE. It is important to note that for ME films the Lifhitz theory of Casimir effect is not relevant but they can be studied in our approach.

The main result of our study on the CE for films in the QED is the following. We have shown that if the CF holds true for thin film from imperfect material, then an interaction of this film with the QED fields can be modeled by photodynamic with the defect action (1) obtained by most general assumptions consistent with locality, gauge invariance and renormalizability of model. Thus, basic principles of QED were essential in our studies of the CE. These principles make it possible to expose new peculiarities of the physics of macroscopic objects in QED and must be taken into account for construction of the models. For plane films we have demonstrated that the CF is not universal and depends on properties of the material represented by the parameter a. For $a \to \infty$ one can obtain the CF for ideal conducting planes. In this case the model coincides with photodynamic considered in [15] with boundary condition $\epsilon^{ijk3}F_{jk} = 0$ $(i = 0, 1, 2)$ on orthogonal to the x_3-axis planes. For sufficiently small a the CF appears to be repulsive. Interaction of plane films with charges and currents generate anomalous magnetic and electric fields which do not arise in classical electrodynamics. The ME materials could be used for observation of phenomena predicted by our model. We hope that the obtained theoretical results can be proven by modern experimental methods.

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